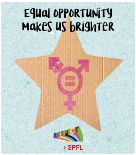
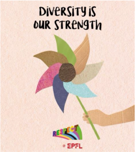
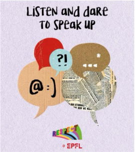
Course 4

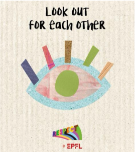
Wednesday, 17 September 2025 09:50

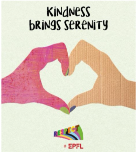
EE557Cours 4 - Doping

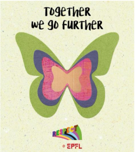

Equal opportunity makes us brighter
RESPECT
EPFL


Diversity is our strength
RESPECT
EPFL


Listen and dare to speak up
?!
@ :)
RESPECT
EPFL


Look out for each other
RESPECT
EPFL


Kindness brings serenity
RESPECT
EPFL


Together we go further
RESPECT
EPFL

Génie Electrique et Electronique
Master program
Prof. Elison Matioli

EE-557
Semiconductor devices I

Doping and carrier concentration

Outline of the lecture

1. Carrier concentration
2. Doping
3. Determining Fermi level

Read Chapter 2 of the reference book

References:

- J. A. del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/)
- C. Kittel, Introduction to Solid State Physics, John Wiley & Sons, 2005

Questions

1. What is a simplified expression for the number of electrons n and holes p?

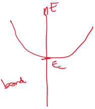
2. What is the position of the Fermi level?

3. What is the np product?

Carrier concentration

$$f(E) = \frac{1}{1 + \exp\left(\frac{E-E_F}{kT}\right)}$$

$$g_c(E) = 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} \cdot \sqrt{E - E_c}$$



effective density of states in conduction band

$$n = \int_{E_c}^{\infty} g_c(E) \cdot f(E)\, dE = N_c \cdot \frac{\cancel{2}}{\cancel{\sqrt{\pi}}} \cdot F_{1/2}\left(\frac{E-E_F}{kT}\right)$$

$$= \int_{E_c}^{\infty} 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} \frac{\sqrt{E-E_c}}{1 + \exp\left(\frac{E-E_F}{kT}\right)} dE$$

↳ Fermi-Dirac integral

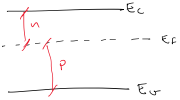
for non degenerate semiconductor:

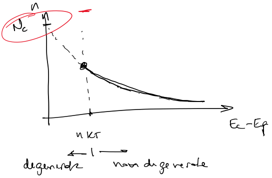
$E_c - E_F \gg n \cdot (kT) \Rightarrow F_{1/2}\left(\frac{E-E_F}{kT}\right) = \frac{\cancel{\sqrt{\pi}}}{\cancel{2}} \cdot \exp\left(-\frac{E-E_F}{kT}\right)$

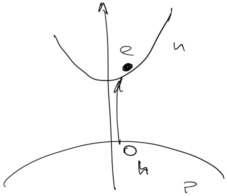
$$\boxed{n = N_c \cdot \exp\left(-\frac{E_c-E_F}{kT}\right)}$$

For the conduction band:

$$n = N_c \exp\left(-\frac{E_c-E_F}{kT}\right)$$







For the valence band

$$p = N_v \exp\left(-\frac{E_F-E_v}{kT}\right)$$

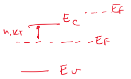
For Si at 300K:

$$N_c = 2 \cdot \left(\frac{2\pi \cdot m^* \cdot kT}{h^2}\right)^{3/2} = \boxed{2.9 \times 10^{19}\ cm^{-3}}$$

$= 2 \cdot \Big(\frac{2\pi \cdot}{\quad}$ .... on moodle

$N_v = 3.1 \times 10^{19}\ cm^{-3}$

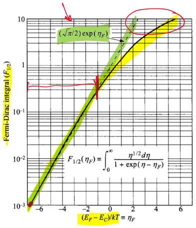
Carrier concentration


Ec
Ef
n.kT
Ef
Ev

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}\left(\frac{E_F - E_C}{kT}\right)$$

where $N_C \equiv 2\left(\frac{2\pi m_{de} kT}{h^2}\right)^{3/2}$

$$F_{1/2}\left(\frac{E_F - E_C}{kT}\right) \equiv F_{1/2}(\eta_F) = \int_{E_C}^{\infty} \frac{[(E - E_C)/kT]^{1/2}}{1 + \exp[(E - E_F)/kT]}\frac{dE}{kT}$$
$$= \int_0^{\infty} \frac{\eta^{1/2}}{1 + \exp(\eta - \eta_F)} d\eta$$

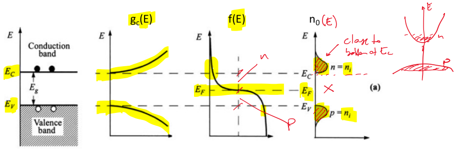


- **Non-degenerate Semiconductors.** By definition, in non-degenerate semiconductors, the doping concentrations are smaller than Nc and the **Fermi levels are more than several kT below Ec**, the Fermi-Dirac integral approaches:

$$F_{1/2}\left(\frac{E_F - E_C}{kT}\right) = \frac{\sqrt{\pi}}{2}\exp\left(-\frac{E_C - E_F}{kT}\right)$$

Carrier concentration

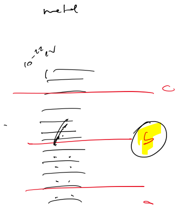
Intrinsic semiconductor (undoped)

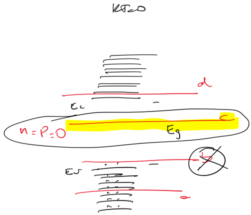

E
Conduction band
E_C
E_g
E_V
Valence band
$g_c(E)$
E
f(E)
E
E_F
n
p
$n_0(E)$
E
E_C
E_F
E_V
$n = n_i$
$p = n_i$
close to bottom of Ec
(a)
E
n
p

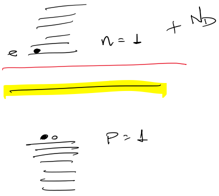
n_o ≡ equilibrium (free) electron concentration in conduction band [cm^{-3}]
p_o ≡ equilibrium hole concentration in valence band [cm^{-3}]

In intrinsic semiconductors:

$n_o = p_o = ni$ n_i ≡ intrinsic carrier concentration [cm^{-3}]


metal
10^{-12} eV
c
S
a


kT=0
d
Ec
c
n = p = 0
Eg
Ev
a



Carrier concentration

Intrinsic semiconductor (undoped)

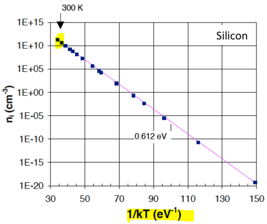
First important result:

$$n_o = p_o = n_i$$

n_i ≡ intrinsic carrier concentration [cm^{-3}]

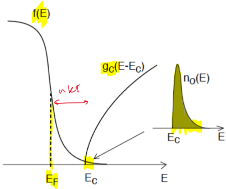
A second important result:

$$n_o p_o = n_i^2$$



Equilibrium ***np* product** in a semiconductor at a certain temperature is a **constant specific to the semiconductor**.

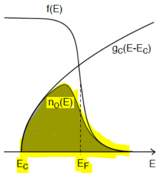
Carrier concentration


f(E)
nkT
$g_C(E-E_C)$
$n_O(E)$
E_C
E
E_F
E_C
E

Non degenerate

$n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$

MB


f(E)
$g_C(E-E_C)$
$n_O(E)$
E_C
E_F
E

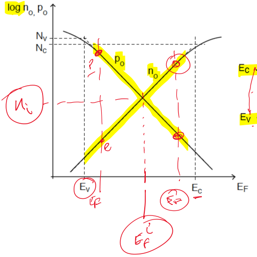
degenerate

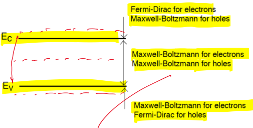
$F_{1/2}$

F-D

Carrier concentration

Summary of carrier statistics depending on E_F location





Fermi-Dirac for electrons
Maxwell-Boltzmann for holes

Maxwell-Boltzmann for electrons
Maxwell-Boltzmann for holes

Maxwell-Boltzmann for electrons
Fermi-Dirac for holes

$$n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right) \quad ①$$

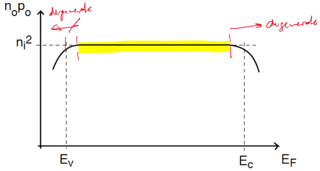
$$p = N_v \cdot \exp\left(-\frac{E_F - E_v}{kT}\right) \quad ②$$

e^{+E_F} e^{-E_F}

Carrier concentration

Summary of carrier statistics depending on E_F location

$n_i = \sqrt{n_o . p_o}$



If E_F is inside bandgap:

$$n_i^2 = n_o p_o \simeq N_c N_v \exp -\frac{E_g}{kT}$$

For a given semiconductor, $n_o p_o$ depends only on T and is independent of precise location of EF . But only if semiconductor is non-degenerate.

Carrier concentration

In intrinsic semiconductor, $n_o = p_o$ and usually fairly small ⇒semiconductor non-degenerate.

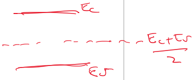
$$(n_i = \sqrt{n_o p_o} = \sqrt{N_c N_v} \exp -\frac{E_g}{2kT}$$

(1) $E_c - E_f = kT \ln (N_c/n)$

(2) $E_f - E_v = kT \ln (N_v/p)$

$$E_f = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \left(\frac{N_v}{N_c} \cdot \cancel{\frac{n}{p}} \right)$$

$$\boxed{E_i \equiv E_f = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \left(\frac{N_v}{N_c} \right)}$$


Ec
$\frac{E_c + E_v}{2}$
Ev

Key questions

- What is doping?
- Where is the Fermi level in a doped semiconductor?

· intrinsic : no doping

· Doping

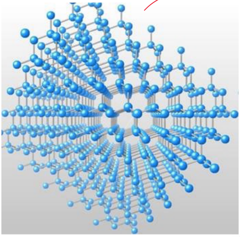
Doping

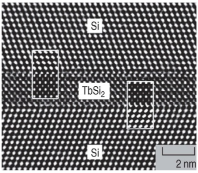
$n = 10^{10}\ cm^{-3}$
$p = 10^{10}\ cm^{-3}$

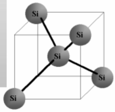
Intrinsic Silicon

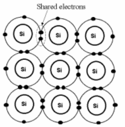
$\rho = 3.2\text{x}10^{3}\ \Omega m$

$5\text{x}10^{22}\ cm^{-3}$




Si
TbSi$_2$
Si
2 nm

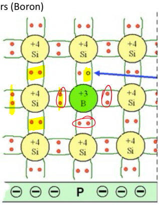

Si
Si
Si
Si
Si

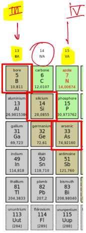

Shared electrons
Si

Doping

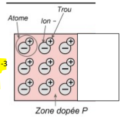
P-type doping with acceptors (Boron)

Silicon p-type:





13 IIIA	14 IVA	15 VA
bore 5 B 10,811	carbone 6 C 12,0107	azote 7 N 14,00674
aluminium 13 Al 26,9815386	silicium 14 Si 28,0855	phosphore 15 P 30,973762
gallium 31 Ga 69,723	germanium 32 Ge 72,61	arsenic 33 As 74,92160
indium 49 In 114,818	étain 50 Sn 118,710	antimoine 51 Sb 121,760
thallium 81 Tl 204,3833	plomb 82 Pb 207,2	bismuth 83 Bi 208,98040
ununtrium 113 Uut [284]	flérovium 114 Fl [289]	ununpentium 115 Uup [288]


Trou
Atome
Ion −
Zone dopée P

Silicon p-type:

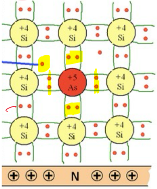
Concentration B = $10^{18}\ cm^{-3}$

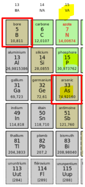
$\rho = 0.04\ \Omega cm = 4\text{x}10^{-4}\ \Omega m$

Doping

n-type

easier to make it free e





13 IIIA	14 IVA	15 VA
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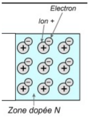
indium 49 In 114,818	étain 50 Sn 118,710	antimoine 51 Sb 121,760
thallium 81 Tl 204,3833	plomb 82 Pb 207,2	bismuth 83 Bi 208,98040
ununtrium 113 Uut [284]	flérovium 114 Fl [289]	ununpentium 115 Uup [288]

Intrinsic

$n = 10^{10}\ cm^{-3} \rightarrow 10^{3}\ \Omega m$

Doped

$n = 10^{18}\ cm^{-3} \rightarrow 10^{-4}\ \Omega m$

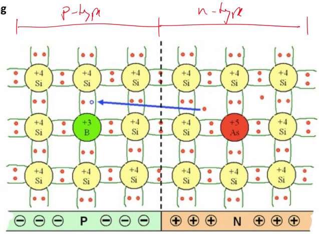

Electron
Ion +
Zone dopée N

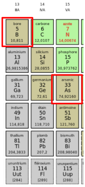
Silicon n-type:

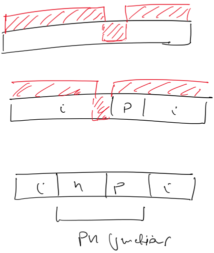
Concentration As = $10^{18}\ cm^{-3}$

$\rho = 0.02\ \Omega cm = 2\text{x}10^{-4}\ \Omega m$

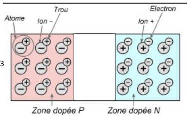
Doping


p-type
n-type
+4 Si
+3 B
+5 As
⊖ ⊖ ⊖ P ⊖ ⊖ ⊖
⊕ ⊕ ⊕ N ⊕ ⊕ ⊕


13 IIIA
14 IVA
15 VA
bore 5 B 10,811
carbone 6 C 12,0107
azote 7 N 14,00674
aluminium 13 Al 26,9815386
silicium 14 Si 28,0855
phosphore 15 P 30,973762
gallium 31 Ga 69,723
germanium 32 Ge 72,61
arsenic 33 As 74,92160
indium 49 In 114,818
étain 50 Sn 118,710
antimoine 51 Sb 121,760
thallium 81 Tl 204,3833
plomb 82 Pb 207,2
bismuth 83 Bi 208,98040
ununtrium 113 Uut [284]
flérovium 114 Fl [289]
ununpentium 115 Uup [288]


i
P
i
i
n
P
i
Pn junction

Silicon p-type:
Concentration B = 10^{18} cm^{-3}
ρ = 0.04 Ωcm = 4x10^{-4} Ωm

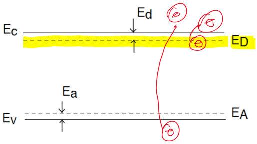

Atome
Ion −
Trou
Electron
Ion +
Zone dopée P
Zone dopée N

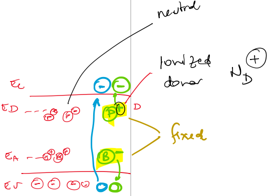
Silicon n-type:
Concentration As = 10^{18} cm^{-3}
ρ = 0.02 Ωcm = 2x10^{-4} Ωm

Carrier concentration

n-type p-type

Representation of donor and acceptor states in energy band diagram:


Ec
Ed
ED
Ea
Ev
EA


neutral
ionized donor
N_D^+
fixed
Ec
ED
D
EA
Ev
P
B

E_D, E_A ~40 −60 meV , for common dopants

$N_D = N_D^+ + n$

Near room temperature, most dopants are ionized:

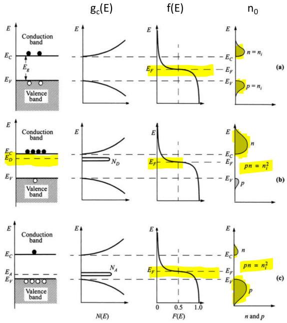
$$N_D^+ \simeq N_D \qquad N_A^- \simeq N_A$$

Typical doping levels: N_A, N_D ~10^{15} −10^{20} cm−3

Carrier concentration

n-type

p-type


$g_c(E)$
f(E)
n_0
E
Conduction band
E_C
E_g
E_V
Valence band
E_F
$n = n_i$
$p = n_i$
(a)
E_D
N_D
n
$pn = n_i^2$
p
(b)
E_A
N_A
(c)
0
0.5
1.0
N(E)
F(E)
n and p

Ionization energies

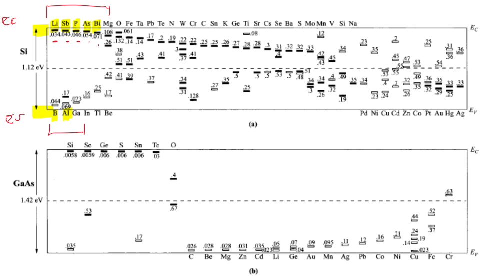
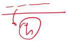
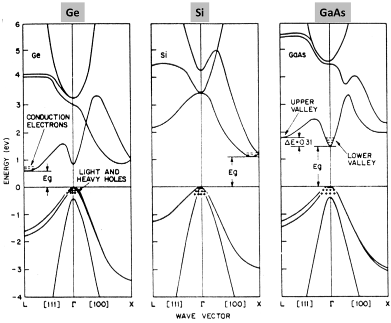


Fig. 10 Measured ionization energies for varies impurities in (a) Si and (b) GaAs. Levels below the gap center are measured from E_V. Levels above the gap center are measured from E_C. Solid bars represent donor levels and hollow boxes represent acceptor levels. (After Refs. 29, 31, 34, and 35.)



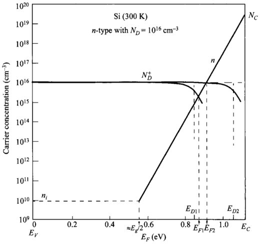
Reminder: valence bands are often degenerated at K = 0 (g_A = 2x2)



Ionization energies

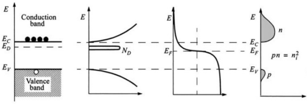
Graphical method:
Determine the Fermi energy level E_F and electron concentration n, when ionization is not complete.

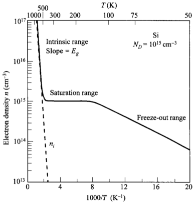
Examples with two different values of impurity levels E_D are shown below:



Ionization energies

Electron density as a function of temperature for a Si sample with donor impurity concentration of 10^{15} cm^{-3}


E
Conduction band
E_C
E_D
E_V
Valence band
E
N_D
E
E_F
E
n
E_C
E_F
E_V
$pn = n_i^2$
p



Key conclusions

Distinct feature of semiconductors: at 0 K, quantum state filling ends up with full band separated from next empty band by ~1-3eV bandgap at around 300 K, some electrons populate next band above bandgap.

System in thermal equilibrium:

- isolated from outside world and in steady state.
- In thermal equilibrium, E_F is independent of position: it is constant!

Occupation probability of quantum systems in thermal equilibrium governed by Fermi-Dirac distribution function:

$$f(E) = \frac{1}{1 + \exp\frac{E-E_F}{kT}}$$

Density of states give us the number of states per volume available at a give energy

In 3D:

$$g_c(E) = 4\pi\left(\frac{2m_{de}^*}{h^2}\right)^{3/2}\sqrt{E-E_c} \qquad E \geq E_c$$

$$g_v(E) = 4\pi\left(\frac{2m_{dh}^*}{h^2}\right)^{3/2}\sqrt{E_v-E} \qquad E \leq E_v$$

Key conclusions

Non-degenerate semiconductor:

$$n_o = N_c \exp \frac{E_F - E_c}{kT}, \qquad p_o = N_v \exp \frac{E_v - E_F}{kT}$$

Intrinsic semiconductor: ideally pure semiconductor.

$$n_o = p_o = n_i = \sqrt{N_c N_v} \exp -\frac{E_g}{2kT}$$

In non-degenerate semiconductor $n_o p_o$ $\quad n_o p_o = n_i^2$

In intrinsic semiconductor, E_F is close to middle of E_g.

In extrinsic semiconductor, E_F location depends on doping level:

n-type non-degenerate semiconductor:

$$n_o \simeq N_D, \qquad E_F - E_c \simeq kT \ln \frac{N_D}{N_c}$$

p-type non-degenerate semiconductor:

$$p_o \simeq N_A, \qquad E_F - E_v \simeq kT \ln \frac{N_v}{N_A}$$